Indian Statistical Institute, Bangalore
B. Math (II)
First semester 2012-2013
Backpaper Examination : Statistics (I)

Date: 01-01-2013 **Maximum Score 75** **Duration: 3 Hours**

1. To establish a standard for parachute design, a researcher recorded the following fill times, in seconds, for 27 standard parachutes, obtained under controlled test conditions.

.59	.38	.47	.43	.44	.37	.43	.37	.27	.54	.39	.89	.48	.52
.51	.49	.38	.38	.23	.44	.40	.36	.33	.82	.51	.44	.37	

(a) Make a stem and leaf plot of these data.

(b) Find the sample mean $\overline{X}$.

(c) Give formula for sample standard deviation s. Do not compute.

(d) Find the sample median M.

(e) Find $100p$-th percentiles for $p = 0.2$ and 0.8.

(f) Find the first and third quartiles.

(g) Draw the box plot and identify the outliers.

(h) For trimming fraction 0.05 obtain the trimmed mean $\overline{X}_{\mathrm{T}}$.

(i) Explain how to obtain the trimmed standard deviation s_{T}. Do not compute.

(j) Between the box plot and the stem and leaf plot what do they tell us about the above data set? In very general terms what can you say about the population from which the data arrived?

$[4 + 2 + 2 + 2 + 4 + 4 + 5 + 3 + 3 + 4 = 33]$

2. The independent random variables $X_1,\ X_2, ..., X_n$ have common distribution specified by

$$P(X \leq x|\alpha, \beta) \quad = \left| \begin{array}{cl} 0 & if\ x < 0 \\ \left(\frac{x}{\beta}\right)^{\alpha} & if\ 0 \leq x \leq \beta \\ 1 & if\ x > \beta \end{array} \right.$$

where α, β are positive. It was found that the length of cuckoos' eggs found in hedge sparrow nests could be modelled with this distribution. Obtain *method of moments estimators* as well as *maximum likelihood estimators* for α, β.

[20]

[PTO]

3. Suppose you can draw a random sample from $U \sim uniform[0,1]$. Explain how you would draw observations on a random variable W that has $Beta\,(m,n)$ distribution, where m and n are positive integers.

[10]

4. Let $(X_1, Y_1), (X_2, Y_2), \cdots, (X_n, Y_n)$ be a random sample from the distribution with following bivariate density function

$$f_{\mathrm{XY}}(x,y) = \frac{1}{2cd} I_{(-d,d)}(x)\, I_{(x^2, x^2+c)}(y)$$

where c, d are positive numbers.

Let $V = \frac{1}{n}\Sigma_{i=1}^{n} X_i$ and $W = \frac{1}{n}\Sigma_{i=1}^{n} Y_i$. Find ρ_{VW}, the correlation coefficient between V and W. What happens as $c \to 0$?

[10 + 2]

5. This amusing classical example is from von Bortkiewicz (1898). The number of fatalities that resulted from being kicked by a horse was recorded for 14 corps of Prussian cavalry over a period of 20 years, giving 280 corps-years worth of data. These data are displayed in the following table. The first column of the table gives the number of deaths per year, ranging from 0 to 4. The second column lists how many times that number of deaths was observed. Thus, for example, in 91 of the 280 corps-years, there was one death.

No. of Deaths per Year	Observed Frequency
0	144
1	91
2	32
3	11
4	2

Do these data come from a *Poisson model?*
Carry out a *chi-square goodness of fit test at level of significance* $\alpha = 0.05$.
Also report the p value.

[15]